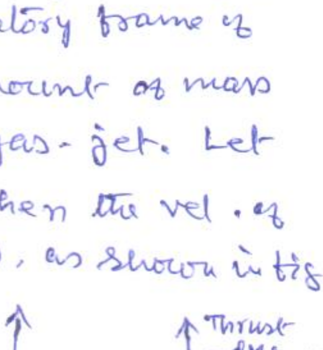
According to Newton's 2nd law, force equals to the rate of change of momentum i.e $F = \dot{p}$. On differentiating the R.H.S, we get $F = m\dot{v} + \dot{m}v$. In other words, if the mass is const. then force equals mass time acceleration - but only if the mass is const. In a rocket, a very appreciable fraction of the mass of the rocket is fuel, which is burned and ejected at a very high rate, so that the mass of the rocket is rapidly diminishing during the motion.

Motion of Rocket.

The rocket is an example of system of variable mass. It works on the principle of conservation of momentum.

Let M (a variable) be the mass of rocket including mass of fuel and V be its vel. at any instant t in laboratory frame of reference. Let in a time interval dt, an amount of mass dM is ejected from the rocket in the form of gas-jet. Let $-u$ be the vel. of gas-jet w.r.to rocket, then the vel. of gas-jet in the laboratory frame will be $(V-u)$, as shown in fig.



The force acting on gas-jet = rate of chang of mass of the rocket due to escape of hot gases x vel.

$$= \frac{dM}{dt}(V-u)$$

From Newton's 3rd Law of motion, this is equal to the thrust on rocket tending to move it in the forward dir$\underline{n}$.

$$\text{Thrust on the rocket} = \frac{dM}{dt}(V-u) \quad \text{--- (1)}$$

Let F_{ext} be the external force acting on the rocket. Here, $F_{ext} = Mg$ (wt. of rocket), then the force acting on the rocket in the forward dirn is $= \frac{dM}{dt}(V-u) - Mg$ --- (2)

From Newton's 2nd law, the force on rocket is $= \frac{d}{dt}(MV)$ --- (3)

Eq$\underline{n}$ (2) = Eq$\underline{n}$ (3), we get

$$\frac{d}{dt}(MV) = \frac{dM}{dt}(V-u) - Mg$$

$$\text{or.} \quad \frac{dV}{dt} = -u\frac{1}{M}\frac{dM}{dt} - g \quad \text{--- (4)}$$

Now integrating eq$\underline{n}$ (4) on both side, we get

$$\int_{v_0}^{v} dv = -u\int_{M_0}^{M} \frac{dM}{M} - g\int_{0}^{t} dt$$

$$\text{or} \quad v - v_0 = -u \log\left(\frac{M}{M_0}\right) - gt$$

$$\therefore \quad v = v_0 + u \log_e\left(\frac{M_0}{M}\right) - gt \quad \ldots (5)$$

Here M_0 = initial mass of rocket and fuel at $t = 0$

M = final " " " at any instant t

u = vel. of gas - jet,

Eqn (5) gives the vel. v of the rocket ~~ate~~ at any instant 't'.

If $g = 0$ and $v_0 = 0$ (initial vel.) : Then eqn (5) gives

$$v = u \log_e\left(\frac{M_0}{M}\right).$$