⇒ Angular Momentum :-

Angular momentum is denoted by $\bar{J}$. It is denoted by vector product of position vector and linear momentum.

i.e $\vec{J} = \vec{r} \times \vec{P}$

Moment :- It is always related to the distance. Ex: Torque = moment of force i.e $\vec{\tau} = \vec{r} \times \vec{F}$

Angular momentum of a particle is defined as moment of linear momentum about a fixed point. It is also defined as vector product of position vector and linear momentum. It is vector quantity.

$$\vec{J} = \vec{r} \times \vec{P} = \vec{r} \times (m\vec{v}) = m(\vec{r} \times \vec{v}) = mrv\sin\theta.$$

The dirn of $\vec{J}$ is perpendicular to the plan containing $\bar{r}$ and $\bar{v}$.

If a body is moving anticlockwise dirn, then dirn of $\vec{J}$ towards you ⊙.

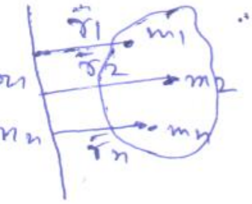
If it is clockwise, then dirn. of $\vec{J}$ away from you ⊗.

SI Unit = m kg ms^{-1} = ~~kg~~ kg m^2s^{-1}

Dimensions = $[L][MLT^{-1}] = [ML^2T^{-1}]$.

⇒ Angular momentum of system of particles: (rigid body) :-

Consider a system of particle which consists of many particle. The position vectors of mass m_1, ~~...~~, m_2, $m_3 \cdots m_n$ are $\bar{r}_1$, $\bar{r}_2$, $\bar{r}_3$ and $\bar{r}_n$ respectively.



[Rigid body :- Distance betn. the two particles dont change]

Angular momentum of system of particle (rigid body) is the vector ~~sum~~ sum of angular momentum of all the particle about the axis of rotation. i.e

$$\bar{J} = \bar{J}_1 + \bar{J}_2 + \cdots + \bar{J}_n$$
$$= (\bar{r}_1 \times \bar{P}_1) + (\bar{r}_2 \times \bar{P}_2) + \cdots + (\bar{r}_n \times \bar{P}_n)$$
$$= \sum_{i=1}^{n} (\bar{r}_i \times \bar{P}_i)$$

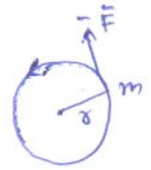
⇒ Relation between angular momentum and angular vel :-

Consider a particle of mass m revolving in a circle of radius r with angular vel. $\bar{\omega}$. Angular mom. of the particle is

$$\vec{J} = \vec{r} \times \vec{p}$$
$$= \vec{r} \times (m\vec{v})$$
$$= m(\vec{r} \times \vec{v})$$
$$= m(\vec{r} \times \vec{r}\omega) \quad [\text{Reln: betn } \vec{v} \text{ and } \vec{\omega} \text{ is } \vec{v} = \bar{r}\vec{\omega}]$$
$$= m r^2 \omega$$
$$\boxed{\vec{J} = I\vec{\omega}}$$

where I is moment of ~~body~~ inertia of the particle about the axis of rotation

Torque :- Torque on a particle is defined as moment of force about the fixed point. It is measured by vector product of position vector and force. Direction is perpendicular to the plane containing $\bar{r}$ and $\bar{F}$.



$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= rF\sin\theta\,\hat{n}$$

. $\vec{\tau}$ is max $\bar{r} \perp \bar{F}$, and minimum $\bar{r} \parallel \bar{F}$.

- SI. Unit :- N m.
- Dimension : $[L][MLT^{-2}] = [ML^2T^{-2}]$

Torque can also be defined as the time rate of change of angular mom.

$$\bar{\tau} = \frac{d\bar{J}}{dt}.$$

According to Newton's 2nd law $F = \frac{dp}{dt}$. Torque $\bar{\tau} = \frac{dJ}{dt}$.

When torque acts on a rotating body about an axis, it produces angular acceleration.

Relation betn. Torque and angular acceleration.

Angular momentum of rigid body

$$\bar{J} = I\bar{\omega}$$

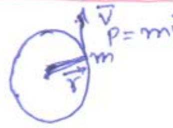
Differentiating w.r.to t.

$$\frac{d\bar{J}}{dt} = I\frac{d\omega}{dt}.$$
$$\boxed{\vec{\tau} = I\bar{\alpha}}, \quad \alpha = \text{angular acceleration.}$$

" Torque is the product of moment of inertia and angular acceleration

⇒ Derivation of relation betn angular momentum & Torque :-

Consider a particle of mass m rotating about a fixed point with vel. $\vec{v}$ and $\vec{r}$ be position vector of the particle.



Angular momentum of the particle is given by

$$\vec{J} = \vec{r} \times \vec{p}$$

Differentiating w.r. to time t

$$\frac{d\bar{J}}{dt} = \frac{d}{dt}(\bar{r}\times\bar{p}) = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\bar{p}}{dt} = \vec{v}\times\vec{p} + \vec{r}\times\bar{F} = \bar{v}\times(m\bar{v}) + \bar{r}\times\bar{F}$$
$$= m(\bar{v}\times\bar{v}) + \vec{r}\times\bar{F}$$
$$= 0 + \bar{r}\times\bar{F}$$
$$\therefore \boxed{\frac{d\bar{J}}{dt} = \vec{\tau}}$$

∴ The rate of change of angular mom. is equal to torque acting on it.

Law of Conservation of Angular mom:—

If net external torque acting on a particle is zero, then the angular momentum of the particle is constant.

We know that $\frac{d\bar{J}}{dt} = \vec{\tau}$

If torque acting on the particle is zero.

$$\frac{d\bar{J}}{dt} = 0$$

$$\boxed{\bar{J} = \text{Const}}.$$

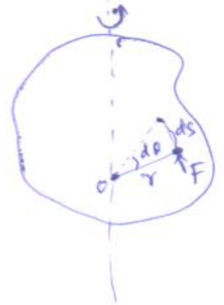
i.e $\boxed{I\bar{\omega} = \text{const}}$

Work done by a Torque:

Consider a rigid body rotating about the axis of rotation.

Let F be the force acting on a particle at a distance r from O producing small displacement ds.

Small work done for displacement ds



$$dw = F\,ds$$

$$= F\,r\,d\theta$$

$$= \tau\,d\theta$$

$\left[\begin{array}{l} Arc = r\theta \\ ds = r\,d\theta \end{array}\right]$

∴ Total work done

$$\int dw = \int \tau\,d\theta$$

$$W = \tau\,\theta$$

∴ This similar to the linear motion. In linear motion workdone $W = FS$.

∴ work done by a torque is the product of torque and angular displacement.