Original research article

Optical solitons in saturable cubic-quintic nonlinear media with nonlinear dispersion

Sudipta Das [a], Kajal Krishna Dey [b], Golam Ali Sekh [c],*

[a] *Department of Physics, Govt General Degree College, Chapra, WB, India*
[b] *Department of Physics, B. B. College, Asansol, WB, India*
[c] *Department of Physics, Kazi Nazrul University, Asansol 713340, India*

A R T I C L E   I N F O

A B S T R A C T

We study an intense-short pulse propagation in a saturable cubic–quintic nonlinear media in the presence of nonlinear dispersion within the framework of an extended variational approach. We derive an effective equation for the pulse width and demonstrate how the saturation due to nonlinearity is achieved in presence of nonlinear dispersion. We find that the nonlinear dispersion can change the pulse width and induce motion in the system. The direction of induced motion depends on the sign of nonlinear dispersion. The pulse is energetically stable at an equilibrium width. A disturbance can, however, induce oscillation in pulse width, the frequency of which is always smaller due to nonlinear dispersion. We check dynamical stability by a direct numerical simulation.

1. Introduction

Studies on the propagation of optical pulse in nonlinear media have been receiving great of attention in the context of solitary-wave-based communication, especially, in femtosecond domain and, consequently, in ultrafast optics (pulse compression) and all-optical switching for last few decades. While propagating several nonlinear phenomena including filamentation, harmonic generation, third-order dispersion, self-steepening and self-frequency shift are associated with the pulse. The observations of such nonlinear phenomena depend crucially on the properties of the medium and the pulse [1–4].

An intense short pulse induces higher-order nonlinearities (HON) in an optical medium. Effects of such pulse are described by various forms of generalized nonlinear Schrödinger equation(GNLSE). The GNLSE is a version of the NLS equation which includes different types of nonlinear terms, some of which have already been realized experimentally. For example, quintic nonlinearity has been obtained in semiconductor doped glasses while septic nonlinearity has been measured in different glasses. It can also be possible to obtain saturation in the nonlinearity in some materials if the intensity of the pulse is relatively high. In semiconductor-doped glasses and organic polymers, however, the nonlinearity becomes saturated even at moderate pulse intensities [5–7].

Self-steepening is one of the effects that commonly arises due to propagation of ultrashort intense pulse in a nonlinear medium. It is related to nonlinear dispersion [8]. In the presence of this effect several attempts have been made to analyse the response of higher-order nonlinearities on the propagation of ultrashort pulse [9–13]. In the recent past, the existence and stability of various types of wave patterns (periodic pattern, bright and dark solitary pulses) have been investigated by Chow et al. It is seen that these modes are stabilized due to competition among different higher-order nonlinearities [14]. The existence of chirped solitary wave solution has been investigated by Triki et al. [15]. Recently, Konar et al. studied the characteristics of chirped solitary pulse in

* Corresponding author.
  E-mail address: skgolamali@gmail.com (G.A. Sekh).

dispersive media with cubic saturable nonlinearity and showed that the pulse broadening can be reduced with increase of saturation limit [16]. The effect of cubic–quintic saturable nonlinearity on the fundamental bright soliton have been studied in higher-order dispersive media in Ref. [17,18].

Our objective in this work is to investigate the properties of bright optical soliton in the presence of nonlinear dispersion and saturable cubic–quintic nonlinearity, specifically, to investigate (i) how does a parameter of soliton reach its saturable limit and (ii) what are the changes in the static and dynamical properties of soliton induced by nonlinear dispersion. In the zero nonlinear dispersion (ND) limit, the system supports stable localized solitary waves due to saturable cubic–quintic nonlinearity. These solitons are dynamically stable. The effect of non-zero ND on the pulse is described by a derivative nonlinear Schrödinger equation (DNSE). We work within the framework of variational approach and find an effective potential for the pulse width in the parameter space. Treating the problem in terms of effective potential in the parameter space is often termed as a potential model. This model is widely used to predict static and dynamical properties of the solitary waves in nonlinear optical media [19–23] and Bose–Einstein condensates [24–26].

In Section 2, we introduce an appropriate mathematical model for the soliton in cubic-quintic nonlinear media and predict the existence of localized bright solution in the limit of negligible nonlinear dispersion. With a view to find the effect of nonlinear dispersion on the solitary wave we work with Ritz optimization procedure in Section 3. We derive equations for different parameters of the solitonic pulse and analyse the effect of saturable nonlinearity. In Section 4 we present the result on the properties of pulse considering nonlinear dispersive effect. We devote Section 5 to make some concluding remarks.

2. Theoretical model

We consider propagation of an intense pulse in optical waveguides and model the evolution of pulse envelope ($\psi(z,\tau)$) by the derivative non-linear Schrödinger equation [14–17,27]

$$i\psi_z = \alpha \frac{d^2\psi}{d\tau^2} + i\gamma|\psi|^2 \frac{d\psi}{d\tau} + \kappa|\psi|^2\psi + \frac{\beta|\psi|^4\psi}{(1+G|\psi|^4)} + \frac{\mu|\psi|^2\psi}{(1+S|\psi|^2)} \tag{1}$$

Here α, γ, κ, β and μ stand for strength of group velocity dispersion, self-steepening, cubic nonlinearity, saturated quintic and cubic nonlinearities respectively. For $0 < G \leq 1$ the 4th term in Eq. (1) can be expanded as: $|\psi|^4\psi - G|\psi|^8\psi + \cdots$. It is called saturable quintic nonlinear term. Similarly, the term for saturated cubic nonlinearity can be expanded for $0 < S \leq 1$. In this model we have considered the dispersive effect up to second-order. This type of dispersion management is feasible in the manufacturing industry by adjusting different parameter/ingredient of an optical fibre.

In order to understand the properties of stationary solution of Eq. (1) in the $\gamma \to 0$ limit, we introduce $\psi(z,\tau) = \phi(\tau)\exp(-i\omega z)$ and write z-independent GNLS equation with saturable nonlinearities

$$\omega\phi = \alpha \frac{d^2\phi}{d\tau^2} + \kappa|\phi|^2\phi + \frac{\beta|\phi|^4\phi}{(1+G|\phi|^4)} + \frac{\mu|\phi|^2\phi}{(1+S|\psi|^2)}, \tag{2}$$

the energy functional for which is given by

$$E[\phi] = \frac{\alpha}{2}\left(\frac{d\phi}{d\tau}\right)^2 + V(\phi) \tag{3}$$

with

$$V(\phi) = -\frac{\beta\tan^{-1}\left(\sqrt{G}\phi^2\right)}{2G^{3/2}} - \frac{(-\beta+G\omega)\phi^2}{2G} + \frac{1}{4}\kappa\phi^4 - \frac{\mu\ln\left(1+S\phi^2\right)}{2S^2} + \frac{\mu\phi^2}{2S}. \tag{4}$$

Here $V(\phi)$ is a double-well shaped potential. It has one positive stationary point and one zero stationary point (left panel Fig. 1). The stationary point at $\phi = 0$ corresponds to the fundamental of solitary-wave solution: $\phi(\tau) \to 0$ as $\tau \to \infty$. Considering $\phi'(0) = 0$ if we take $\phi(0) \geq \phi$ for solving Eq. (2), we can obtain different modes of solutions [28]. With a view to confine our attention to fundamental mode of solution we take $\phi = \phi(0) + \Delta\phi$ and solve of Eq. (2) numerically. The result is displayed in the right panel of Fig. 1. We see that the NLS with saturable cubic-quintic nonlinearity (SCQNL) can support bright type solitary profile. It is relevant to note that for $\gamma \neq 0$, the existence of chirped solitary profile in the presence of unsaturated nonlinearity has recently been analysed in Ref. [14,15].

3. Variational formulation

We have seen that the stationary solution of Eq. (1) supports a single hump bright soliton. In order to understand the effect of nonlinear dispersion on the dynamics of optical pulse propagating in a cubic–quintic saturable nonlinear medium we consider variational approach. To begin with we follow inverse variational method and write Lagrangian density $\mathcal{L}$ for Eq. (1) as

$$\mathcal{L} = i(\psi\psi_z^* - \psi^*\psi_z) + \frac{1}{2}i\gamma|\psi|^2(\psi_\tau\psi^* - \psi_\tau^*\psi) - 2\alpha\left|\frac{d\psi}{d\tau}\right|^2 + \kappa|\psi|^4 + \frac{2\beta}{G}|\psi|^2 - \frac{2\beta\tan^{-1}(\sqrt{G}|\psi|^2)}{G^{3/2}} + \frac{2\mu}{S}|\psi|^2 - \frac{2\mu}{S^2}\ln(1+S|\psi|^2) \tag{5}$$

and we consider

$$\psi(z,\tau) = B\,\mathrm{sech}\left[\frac{\tau-\tau_0}{w}\right] \times \exp i\left[v(\tau-\tau_0) + \frac{q}{2w}(\tau-\tau_0)^2 + \sigma\right] \tag{6}$$
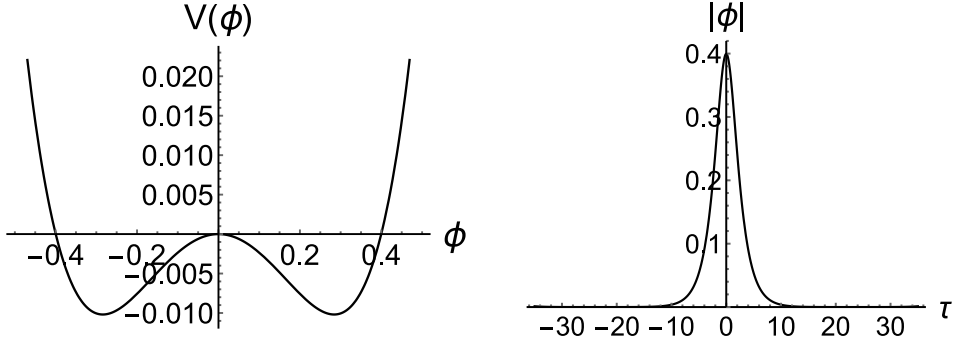
Fig. 1. Left panel: The potential $V(\phi)$ as a function of ϕ for $\kappa = 4$, $\mu = 2$, $\beta = 3$ and $\omega = 0.5$ for bright type profile in a saturable quintic and cubic nonlinear media with $G = 0.27$ and $S = 0.27$. Right panel: Stationary solution of NLS with saturable quintic nonlinearity for $\alpha = 1.7$ with the initial condition $\phi(0) = 0.399605$ and $\phi'(0) = 0$.

as a trial solution of Eq. (1) for the bright density profile with the norm $P = \int |\psi(z, \tau)|^2 \, d\tau = 2w|B|^2$. Here, B, τ_0 and w represent respectively the complex amplitude, central position, pulse width of the ψ envelope. Other parameters, namely, σ, v and q indicate the phase, velocity (centre of the soliton) and frequency chirp respectively. In the variational analysis, we allow all these parameters to vary with z with view to catch effects of the system [29].

Inserting Eq. (6) in Eq. (5) and then integrating the resulting equation with respect to τ from $-\infty$ to $+\infty$ we obtain the following averaged Lagrangian density

$$\langle \mathcal{L} \rangle = 2i \left(B \frac{dB^*}{dz} - B^* \frac{dB}{dz} \right) w + 2|B|^2 \left(-2vw \frac{d\tau_0}{dz} + \frac{\pi^2}{12} w^2 \frac{dq}{dz} - \frac{\pi^2}{12} qw \frac{dw}{dz} + 2w \frac{d\sigma}{dz} \right) - \frac{4}{3} \gamma |B|^4 wv$$
$$- 4\alpha |B|^2 \left(\frac{1}{3w} + v^2 w + \frac{\pi^2}{12} q^2 w \right) + \frac{4}{3} \kappa |B|^4 w + \frac{4\beta}{G} |B|^2 w - \beta M |B|^{2(2n-1)} w + \frac{4\mu}{S} |B|^2 w - \mu N |B|^{2n} w, \tag{7}$$

where

$$M = \frac{2}{G^{3/2}} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} G^{\frac{2n-1}{2}} \left(M_1 + M_2 \right)$$

with

$$M_1 = \frac{(-1 + 3n)\sqrt{\pi}\Gamma(-1 + 2n)}{2\Gamma(\frac{1}{2} + 2n)} \quad \text{and} \quad M_2 = 2^{(-3+4n)} \frac{\Gamma(-1 + 2n)\Gamma(1 + 2n)}{\Gamma(4n)}$$

and

$$N = \frac{2}{S^2} \sum_{n=1}^{\infty} \frac{S^n (-1)^{n+1} \sqrt{\pi}\Gamma(n)}{n\Gamma(n + \frac{1}{2})}.$$

While calculating averaged Lagrangian in Eq. (7), we expand the terms [Eq. (6)] arising from cubic and quintic saturable nonlinearities. Understandably, the value of n gives the order of nonlinearity.

3.1. Evolution equations in parameter space

From the vanishing conditions of the variational derivatives $\frac{\delta\langle\mathcal{L}\rangle}{\delta B}$, $\frac{\delta\langle\mathcal{L}\rangle}{\delta B^*}$, $\frac{\delta\langle\mathcal{L}\rangle}{\delta \tau_0}$, $\frac{\delta\langle\mathcal{L}\rangle}{\delta w}$, $\frac{\delta\langle\mathcal{L}\rangle}{\delta v}$, $\frac{\delta\langle\mathcal{L}\rangle}{\delta q}$ and $\frac{\delta\langle\mathcal{L}\rangle}{\delta \sigma}$ we obtain the following equations

$$- 4iw \frac{dB^*}{dz} - 2iB^* \frac{dw}{dz} + 4B^* vw \frac{d\tau_0}{dz} - \frac{\pi^2}{6} B^* w \left(w \frac{dq}{dz} - q \frac{dw}{dz} \right) - 4B^* w \frac{d\sigma}{dz} + \frac{8}{3} \gamma v w |B|^2 B^* + \frac{4}{3} \alpha \frac{B^*}{w}$$
$$+ 4\alpha B^* v^2 w + \frac{\pi^2}{3} \alpha B^* q^2 w - \frac{8}{3} \kappa |B|^2 B^* w - \frac{4\beta}{G} B^* w + \beta M(2n-1)|B|^{2(2n-2)} B^* w - \frac{4\mu}{S} B^* w + \mu N n |B|^{2(n-1)} B^* w = 0, \tag{8}$$

$$4iw \frac{dB}{dz} + 2iB \frac{dw}{dz} + 4Bvw \frac{d\tau_0}{dz} - \frac{\pi^2}{6} Bw \left(w \frac{dq}{dz} - q \frac{dw}{dz} \right) - 4Bw \frac{d\sigma}{dz} + \frac{8}{3} \gamma v w |B|^2 B + \frac{4}{3} \alpha \frac{B}{w}$$
$$+ 4\alpha B v^2 w + \frac{\pi^2}{3} \alpha B q^2 w - \frac{8}{3} \kappa |B|^2 Bw - \frac{4\beta}{G} Bw + \beta M(2n-1)|B|^{2(2n-2)} Bw - \frac{4\mu}{S} Bw + \mu N n |B|^{2(n-1)} Bw = 0, \tag{9}$$

$$\frac{d}{dz} \left[|B|^2 wv \right] = 0, \tag{10}$$

$$- \frac{\pi^2}{6} \frac{d}{dz} \left(|B|^2 w \right) q - \frac{\pi^2}{2} |B|^2 w \frac{dq}{dz} + 2i \left(B^* \frac{dB}{dz} - B \frac{dB^*}{dz} \right) + 4|B|^2 v \frac{d\tau_0}{dz} + \frac{\pi^2}{6} |B|^2 q \frac{dw}{dz} - 4|B|^2 \frac{d\sigma}{dz} + \frac{4}{3} \gamma |B|^4 v - \frac{4}{3} \alpha \frac{|B|^2}{w^2}$$

$$+ 4\alpha |B|^2 v^2 + \frac{\pi^2}{3} \alpha |B|^2 q^2 - \frac{4}{3} \kappa |B|^4 - \frac{4\beta}{G} |B|^2 + \beta M |B|^{2(n-1)} - \frac{4\mu}{S} |B|^2 + \mu N |B|^{2n} = 0, \tag{11}$$

$$\frac{d\tau_0}{dz} + \frac{1}{3} \gamma |B|^2 + 2\alpha v = 0, \tag{12}$$

$$\frac{dw}{dz} = -2\alpha q, \tag{13}$$

$$\frac{d}{dz} [w|B|^2] = 0, \tag{14}$$

The last equation can also be obtained by multiplying Eq. (8) by B and Eq. (9) by B^* and then subtracting the resulting equations. However, adding Eq. (8)×B and Eq. (9)×B^* and then using Eq. (11) we get

$$w \frac{dq}{dz} = -\frac{4}{\pi^2} \gamma v |B|^2 - \frac{8\alpha}{\pi^2 w^2} + \frac{4}{\pi^2} \kappa |B|^2 - \frac{6\beta}{\pi^2} (n-1) M |B|^{4(n-1)} - \frac{3\mu}{\pi^2} (n-1) N |B|^{(2n-2)}. \tag{15}$$

This is the so-called chirp equation. This equation in conjunction with Eq. (12) leads to

$$\frac{d^2 w}{dz^2} = \frac{16\alpha^2}{\pi^2 w^3} - \frac{8\alpha\kappa}{\pi^2} \frac{|B|^2}{w} + \frac{8\alpha\gamma}{\pi^2} v \frac{|B|^2}{w} + \frac{12\alpha\beta}{\pi^2} M(n-1) \frac{|B|^{4(n-1)}}{w} + \frac{6\alpha\mu}{\pi^2} (n-1) N \frac{|B|^{2(n-1)}}{w}. \tag{16}$$

Eq. (14) implies that $w|B|^2$ is a constant (say, E_0) which is related to norm (P) of the system. The Eq. (10) immediately demands that v is constant. However, the presence of nonlinear dispersion ($\gamma \neq 0$) in Eq. (12) indicates that the velocity of the centre of motion can be changed for any non-zero value of pulse width due to nonlinear dispersion.

4. Dynamics of optical soliton

In the previous section, we have derived equations for different parameters and their dependence on the parameters of the system. It is interesting to find the values of system's parameters which can permit stable soliton solution. This can be achieved by analysing the effective potential model [10,29]. In order of this, we write an effective equation in terms of w from Eq. (16) as

$$\frac{d^2 w}{dz^2} = \frac{16\alpha^2}{\pi^2 w^3} - \frac{8\alpha\kappa}{\pi^2} \frac{E_0}{w^2} + \frac{8\alpha\gamma}{\pi^2} v \frac{E_0}{w^2} + \frac{12\alpha\beta}{\pi^2} M(n-1) \frac{E_0^{2(n-1)}}{w^{2n-1}} + \frac{6\alpha\mu}{\pi^2} N(n-1) \frac{E_0^{n-1}}{w^n}. \tag{17}$$

For the convenience of analysis, we introduce the normalized pulse width $y(z) = \frac{w(z)}{w_0}$ and write effective potential from Eq. (17) as

$$\Pi(y) = \frac{S_1}{y^2} - \frac{S_2}{y} + \frac{S_3}{y} + \sum \frac{\Omega_n}{y^{2n-2}} + \sum \frac{\Gamma_n}{y^{n-1}} - \left(S_1 - S_2 + S_3 + \sum \Omega_n + \sum \Gamma_n \right), \tag{18}$$

where

$$S_1 = \frac{8\alpha^2}{\pi^2 w_0^4}, S_2 = \frac{8\alpha\kappa E_0}{\pi^2 w_0^3}, S_3 = \frac{8\alpha\gamma v E_0}{\pi^2 w_0^3} \tag{19}$$

and

$$\Omega_n = \frac{6\alpha\beta}{\pi^2} M \frac{E_0^{2n-2}}{w_0^{2n}}, \quad \Gamma_n = \frac{6\alpha\mu}{\pi^2} N \frac{E_0^{n-1}}{w_0^{n+1}}. \tag{20}$$

In Fig. 2, we plot the variation of effective potential $\Pi(y)$ with y for different values of n to find the effect of higher order terms arising from saturable nonlinearity. Clearly, the $\Pi(y)$ has a minimum V_m for the chosen values of system's parameters and n. The values of width (w_m) corresponding to V_m gives stable pulse width. We know that for $n = 1$, the effects of higher order nonlinearity is negligible and, therefore, the black solid curve gives effective potential for the fundamental soliton solution. With the increase of n value, the effect of higher order nonlinearity comes into play. As a result, energy and width of the soliton start to vary due to interplay among dispersion, Kerr nonlinearity and saturable quintic non-linearity. More specifically, the value of w_m and V_m decrease with the increase of n. In each case, the effect of nonlinear dispersion makes the potential less deeper (right panel of Fig. 2). However, the effect of nonlinear dispersion is dominating in absence of saturable nonlinearities (SNs). Thus the SNs can play an important role in stable pulse propagation.

To illustrate the insight of saturable limit in present context we display in Fig. 3 the variation of w_m and V_m with n for (i) $\gamma = 0$ and (ii) $\gamma \neq 0$. The value of w_m increases with the order n of saturable nonlinearities. However, the growth of w_m stops at $n = 9$. We termed this region as unsaturated region. In this region the dispersive effect dominates over the nonlinear effects resulting a wider pulse. However, in the saturation region ($n \geq 9$) the nonlinear effect can compensate the dispersive effect and thus the pulse width
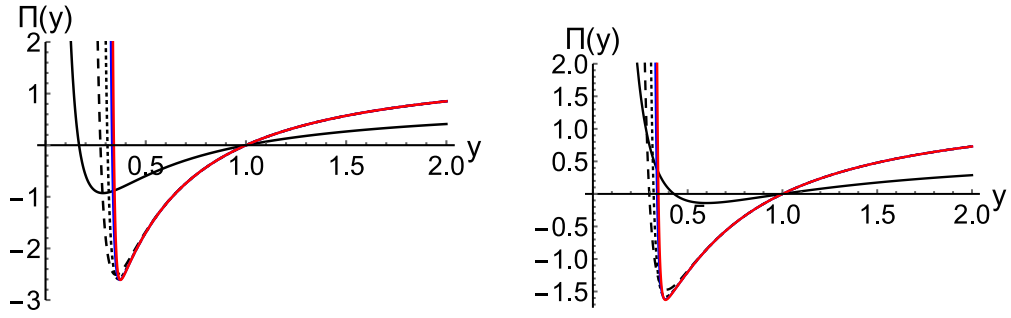
Fig. 2. Plot of effective potential with pulse width in saturable nonlinear media for $n = 1$(black), $n = 5$(black dash), $n = 9$(black dotted), $n = 15$(blue) and $n = 17$(red). In all the panels we have taken $\alpha = 1.72$, $\beta = 3$, $\kappa = 4$, $\mu = 2$, $w_0 = 2$ and $G = S = 0.27$. Left panel gives the curves for $\gamma = 0$ and $v = 0$ while we take $\gamma = 0.5$ and $v = 1.25$ in right panel. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
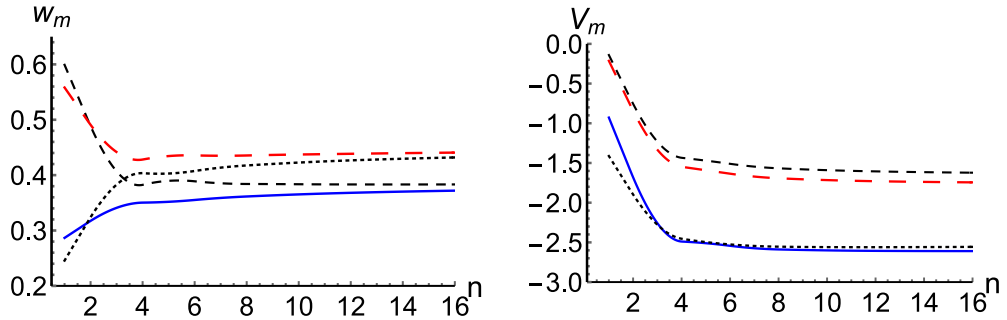

Fig. 3. Left panel: Effective width corresponding to the potential minimum with the order of quintic saturable nonlinearity. Right panel: Minimum value of effective potential with the order of saturable cubic-quintic nonlinearity. In all the panels we have taken $\alpha = 1.72$, $\beta = 3$, $\kappa = 4$, $v = 1.25$, $w_0 = 2$ and $G = S = 0.27$. Here (i) $E_0 = 1.5$, $\gamma = 0$ (solid blue), (ii) $E_0 = 1.5$, $\gamma = 0.5$ (black dashed), (iii) $E_0 = 1.75$, $\gamma = 0$ (black dotted) and (iv) $E_0 = 1.75$, $\gamma = 1.25$ (red dashed). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
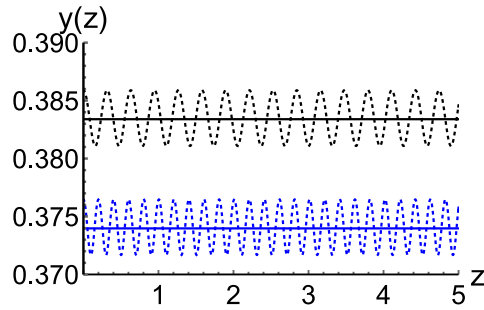

Fig. 4. Variation of effective width as a function of z in cubic-quintic saturable nonlinear media with nonlinear dispersion. Here we have taken $\alpha = 1.72$, $\beta = 3$, $\kappa = 4$, $G = S = 0.27$, $\mu = 2$, $E_0 = 1.5$ and $w_0 = 2$. (i) $\gamma = 0$ and $v = 0$ (blue curve), (ii) $\gamma = 0.5$ and $v = 1.25$ (black curve). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

approaches a constant value. This effect can be controlled by E_0. More specifically, the value of saturated pulse width or energy for $E_0 = 1.75$ is higher than that for $E_0 = 1.5$. Therefore, without loss of generality, we truncate the series arising due to saturable cubic–quintic nonlinearity at $n = 19$ and solve $d^2y/dz^2 + d\Pi(y)/dy = 0$ numerically taking $y(0) = w_m$ and $y'(0) = 0$. The variation of normalized pulse width with z is shown Fig. 4. We see that the pulse width remains unchanged. The value of width for $\gamma \neq 0$ slightly differs from that for $\gamma = 0$. However, if the initial pulse width is taken slightly larger than the equilibrium value, pulse width starts to oscillate in both cases. Their frequencies of oscillations (v) are not equal. The nonlinear dispersion reduces the value of v.

We perform exact numerical simulation of Eq. (1) in the saturated nonlinear regime with initial conditions $\psi(0, \tau) = B(0)\operatorname{sech}[\tau/w(0)]$ and $\psi'(z)|_{z=0} = 0$. Here we fixed initial values of $B(0)$ and $w(0)$ at the minimum of the effective potential and calculated density profile at $z = 100$ for $\gamma = 0$ (left panel, Fig. 5). We notice that the position and width of the density profile remains unchanged with z for $\gamma = 0$ as predicted by the variational approach. This indicates that the pulse is dynamically stable. Interestingly, in the presence ($\gamma \neq 0$) of nonlinear dispersion (ND), the soliton starts to move with a constant velocity keeping its shape unchanged (right panel of Fig. 5). The direction of movement depends on the sign of γ. For a negative value of γ it moves
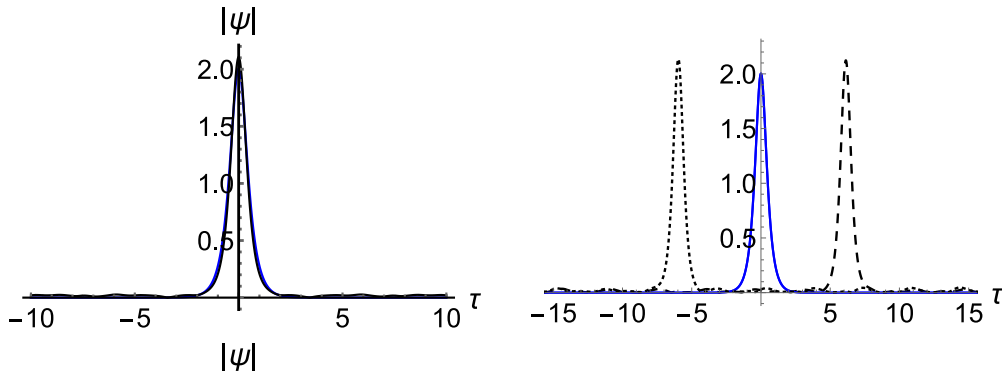
Fig. 5. Exact numerical solution of Eq. (1) for (i) $\gamma = 0$ (left panel) and (ii) $\gamma = \pm 0.5$ (right panel). In left panel, the blue curve represents density profile calculated at $z = 100$ while the black curve gives variational result. In right panel the blue curve gives density profile for $\gamma = 0$ while dotted and dashed curve give density profiles for $\gamma = 0.5$ (left side) and $\gamma = -0.5$ (right side) calculated at $z = 8$ from exact numerical simulation of Eq. (1). In all the panels we have taken $\alpha = 1.72$, $\beta = 3$, $\mu = 2$, $\kappa = 4$ and $G = S = 0.27$.

towards the right (dashed curve) while it moves towards the left (dotted curve) if γ is positive. However, the profile remains stable even at a larger value of z. This result is consistent with the variational prediction in Eq. (12).

5. Conclusions

Pulse propagation in an optical medium depends on nonlinear and dispersive responses of the medium. Interestingly, the modern technology has paved the way of engineering materials with manageable nonlinearity and dispersion in the manufacturing industry by adjusting different parameters/ingredients of the system. In this work we have discussed optical pulse propagation in saturable cubic-quintic nonlinear media with nonlinear dispersion.

We adopted an extended variational method and derived an effective potential for pulse width based on a suitable chosen trial solution. We estimated the trial solution from the shape of the stationary pulse in the zero nonlinear dispersion region. In the variational technique we allow different parameters to vary in order to catch the effect of nonlinear dispersion. Based on a potential model we have examined that energy and width of the soliton attain a limiting value after a certain order of nonlinear terms. However, the saturation limit depends on the power of pulse. We have shown that in the unsaturated regime the cubic-quintic nonlinear medium with nonlinear dispersion supports relatively wider solitons. These solitons are stable. If the soliton is disturbed then its width oscillates about its equilibrium value. The oscillation frequency is affected by the strength of nonlinear dispersion.

We envisaged a direct numerical simulation for the dynamics of soliton both in presence and absence of nonlinear dispersion (ND). In the zero ND regime, the characteristics and position of solitons are found to be unchanged for a long time implying that it is dynamically stable. However, due to nonlinear dispersion the soliton starts to move with a constant velocity. The direction of the movement is determined by sign of the nonlinear dispersion term. A similar dynamics can be realized due Raman effect [18].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

References

[1] G.P. Agrawal, Nonlinear Fiber Optics, fifth ed., Academic Press, Elsevier, ISBN: 9780123970237, 2012.
[2] L.F. Mollenauer, J.P. Gordon, Solitons in Optical Fibers: Fundamentals and Applications, first ed., Academic Press, Elsevier, ISBN: 9780125041904, 2006.
[3] S.P. Singh, N. Singh, Nonlnear effects in optical fibers: Origin, Manag. Appl. PIER 73 (2007) 249275.
[4] Mario F.S. Ferreira, Nonlinear effects in optical fibers: Limitation and possibility, J. Nonlinear Opt. Phys. Mater. 17 (2008) 23.
[5] L.H. Acioli, et al., Femtosecond dynamics of semiconductor-doped glasses using a new source of incoherent light, Appl. Phys. Lett. 56 (1990) 2279.
[6] B.L. Lawrence, G.I. Stegeman, Two-dimensional bright spatial solitons stable over limited intensities and ring formation in polydiacetylene para-toluene sulfonate, Opt. Lett. 23 (1998) 591.
[7] M.F. Ferreira, Nonlinear Effects in Optical Fibers, first ed., Wiley, ISBN: 978-0-470-46466-3, 2011.
[8] F.M. Mitschkem, L.F. Mollenauer, Discovery of the soliton self-frequency shift, Opt. Lett. 11 (1986) 659.

[9] L. Bergé, et al., Dynamics of localized solutions to the Raman-extended derivative nonlinear Schrö dinger equation, J. Phys. A: Math. Gen. 29 (1996) 3581.

[10] Debabrata Pal, Sk. Golam Ali, B. Talukdar, Embedded solitons in the third-order nonlinear Schrodinger equation, Phys. Scr. 77 (2008) 065401; Sudipta Das, Golam Ali Sekh, Dynamics of compressed optical pulse in cubic-quintic media, Fiber Integr. Opt. 39 (2020) 122.

[11] A. Choudhuri, K. Porsezian, Dark-in-the-Bright solitary wave solution of higher-order nonlinear Schrödinger equation with non-Kerr terms, Opt. Commun. 285 (2012) 364.

[12] H. Triki, Abdul-Majid Wazwaz, Soliton solutions of the cubic-quintic nonlinear Schrödinger equation with variable coefficients, Rom. J. Phys. 61 (2016) 360.

[13] R. Radhakrishnan, A. Kundu, M. Lakshmanan, Coupled nonlinear Schrödinger equations with cubic-quintic nonlinearity: Integrability and soliton interaction in non-Kerr media, Phys. Rev. E 60 (1999) 3314.

[14] K.W. Chow, C. Rogers, Localized and periodic wave patterns for a nonic nonlinear Schrödinger equation, Phys. Letts. A 377 (2013) 2546.

[15] Houria Triki, et al., Chirped solitary pulses for a nonic nonlinear Schrödinger equation on a continuous-wave background, Phys. Rev. A 93 (2016) 063810.

[16] S. Konar, Anjan Biswas, Chirped optical pulse propagation in saturating nonlinear media, Opt. Quantum Electron. 36 (2004) 905.

[17] J. Atangana, et al., Cubic-quintic saturable nonlinearity effects on a light pulse strongly distorted by the fourth-order dispersion, J. Modern Opt. 60 (2013) 292.

[18] Ping Wang, Tao Shang, Li Feng, Yingjie Du, Solitons for the cubic-quintic nonlinear Schrödinger equation with Raman effect in nonlinear optics, Opt. Quantum Electron. 46 (2014) 1117.

[19] D. Anderson, Variational approach to nonlinear pulse propagation in optical fibers, Phys. Rev. A 27 (1983) 3135.

[20] D. Pal, Sk. Golam Ali, B. Talukdar, Embedded soliton solutions: A variational study, Acta Phys. Polon. A 133 (2008) 677.

[21] Boris A. Malomed, Pulse propagation in nonlinear optical fiber with periodically modulated dispersion: Variational approach, Opt. Commun. 136 (1997) 313.

[22] Sk. Golam Ali, D. Pal, S.K. Roy, B. Talukdar, Application of variational calculas to propagation of coupled pulses in optical fibers, Czech. J. Phys. 56 (2006) 217–316.

[23] Maitrayee Saha, Samudra Roy, Shailendra K. Varshney, Variational approach to study soliton dynamics in a passive fiber loop resonator with coherently driven phase-modulated external field, Phys. Rev. E 100 (2019) 022201.

[24] Sekh Golam Ali, Bouncing dynamics of Bose–Einstein condensates under the effects of gravity, Phys. Letts. A 381 (2017) 852.

[25] Yongshan Cheng, Rongzhou Gong, Hong Li, Dynamics of two coupled Bose–Einstein Condensate solitons in an optical lattice, Opt. Express 14 (2006) 3594.

[26] Sk. Golam Ali, B. Talukdar, Coupled matter-wave solitons in optical lattices, Ann. Phys., NY 324 (2009) 1185.

[27] Jincheng Shi, Jianhua Zeng, Boris A. Malomed, Suppression of the critical collapse for one-dimensional solitons by saturable quintic nonlinear lattices, Chaos 28 (2018) 075501.

[28] Zlatko Jovanoski, David R. Rowland, Variational analysis of solitary waves in a homogeneous cubic-quintic nonlinear medium, J. Modern Opt. 48 (2001) 1179.

[29] D. Pal, Sk. Golam Ali, B. Talukder, Embedded solitons in the third-order nonlinear Schrödinger equation, Phys. Scr. 77 (2008) 065401.